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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2015

FIRST YEAR [BATCH 2015-17]

MATHEMATICS

Date : 31/12/2015

Time : 11 am – 1 pm

Paper : IV

Full Marks : 50

Answer all the questions. Maximum you can score is 50.

1. Call a subset of $\mathbb{R}^2$ 'radially open' if it contains an open line segment in each direction about each of its points. Show that
   a) The collection of all radially open sets is a topology for $\mathbb{R}^2$. [The plane $\mathbb{R}^2$ with this topology is called the 'radial plane'] [3]
   b) Any circle is a closed discrete subset of the radial plane. [3]
   c) The radial plane is separable but neither 1st countable nor Lindelöf. [3+3+1]

2. a) Suppose $\mathbb{R}_\ell$ denotes the Sorgenfrey line. Find a continuous map $f:\mathbb{R}_\ell \to \mathbb{R}$ such that $f:\mathbb{R}\to\mathbb{R}$ is not continuous. [3]
   b) Let ψ be the family of all maps having the property of f given in 2(a). Find the cardinal number of ψ. [4]

3. Show that the map $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x) = x + x^5\ \forall\, x\in\mathbb{R}$ is a homeomorphism. [5]

4. Prove that any regular Lindelöf space is normal. [6]

5. Prove that any two disjoint compact sets in a Hansdorff space can be strongly separated. [5]

6. Suppose X is a metric space such that each function is C(X) is uniformly continuous. Prove that X is complete. [4]

7. a) Let X and Y be two locally compact Hansdorff spaces. If $X \cong Y$ then prove that $X^+ \cong Y^+$ where X^+ and Y^+ denote respectively the one-point compactifications of X and Y. [4]
   b) Prove that the one point compactification of $\mathbb{N}$ is homeomorphic to $\left\{0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$. [3]

8. Prove that a connected separable metric space with more than one point has cardinal c. [6]

9. a) Prove that a product of compact spaces is compact. [4]
   b) Show that $\mathbb{R}_\ell \times \mathbb{R}_\ell$ is not Lindelöf where $\mathbb{R}_\ell$ denotes the Sorgenfrey line. [3]

10. a) If X and Y are two sets such that Card $X \le$ Card Y and Card $Y \le$ Card X then prove that Card X = Card Y. [6]
    b) Let α be any Cardinal number. Prove that there exists a Cardinal number β such that $\alpha < \beta$. [4]

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